Last revision: May 2006

Consent and Exchange

Oren Bar-Gill† and Lucian Arye Bebchuk‡

Abstract

This paper considers the basic legal principle under which the consent of both parties is needed for an exchange in which an asset or a service is provided by a "seller" to a "buyer." We compare the effects of this "mutual consent" regime to those of the alternative "restitution" regime, under which agents may confer a benefit on a (forced) "buyer" and thereby become entitled to receive from the buyer a payment equal to the buyer's benefit as estimated by a court. The analysis focuses on the beneficial effects that the mutual consent regime has on *ex ante* actions and investments. When courts are imperfectly informed about the value of an exchange to the parties, the restitution rule would have large *ex ante* efficiency costs *even when courts' errors are unbiased*. With such imperfect information by courts, the restitution rule would induce inefficient investments – by buyers in avoiding opportunities for an exchange and by sellers in creating such opportunities -- and indeed might well lead to a massive withdrawal of potential buyers from the market. The symmetric "pricing" regime, the "takings" regime, under which a "buyer" may take an asset from a (forced) "seller" provided that the buyer pay the seller's cost as estimated by a court, is inferior to the mutual consent regime for similar reasons.

JEL classification: C72, C78, D23, K10, K11, K12
Key words: ownership, property rights, exchange, bargaining.

---

† Assistant Professor of Law, NYU School of Law.
‡ William J. Friedman Professor of Law, Economics, and Finance, Harvard Law School; Research Associate, National Bureau of Economic Research. We thank Omri Ben-Shahar, Bernie Black, Hanoch Dagan, Assaf Hamdani, Sharon Hannes, Oliver Hart, Louis Kaplow, Eric Rasmusen, Steve Shavell, Chris Snyder, Kathy Spier, workshop and conference participants at Harvard University, Tel-Aviv University and the 2004 NBER Summer Institute for their helpful comments. We have benefited from the financial support of the John M. Olin Center for Law, Economics, and Business at Harvard Law School.

1 Introduction

Exchanges – transfers of value from a "seller" to a "buyer" – generally require the mutual consent of both sides to the exchange. Much work in economics takes this feature of the world for granted. But this feature is not an essential corollary of the concept of exchange. Rather, it is a product of how the legal system has chosen to regulate exchange. This paper analyzes the critical role that the mutual consent principle performs in the operation of an economy.

Specifically, we contrast the mutual consent principle with the alternative "Restitution" Rule. Under this rule, a potential "seller" may elect to confer a value on another party – say, transfer an asset or provide a service – and thereby, absent an agreement between the parties to indicate otherwise, become entitled to a payment from the other party equal to the value of the asset or service to the other party as estimated by a court. That is, under this rule, potential sellers have a "put" option entitling them to sell an asset or a service to another party with an exercise price equal to the court-estimated value of such transfer to the other party. In contrast, under the Mutual Consent Rule, since potential sellers cannot unilaterally become entitled to any payment, they can be viewed as having a put option with an exercise price of zero.

There are circumstances in which the legal system does impose a Restitution rule. They generally arise when there are impediments to an agreement between the parties. If B is in a comma or drowning (situations in which entering an agreement would be rather difficult) and S takes the expense to help B, S will generally become entitled to a payment by B even though B did not agree to make such a payment. Similarly, under a "common fund" doctrine, someone who makes expenditures to advance a certain interest of a class might become entitled to payments from the members of the class; under this doctrine, for example, plaintiffs in a class action might become entitled to get payments from fellow members in the event that they benefit from the litigation. However, these are exceptions. In general, a potential seller needs to get a potential buyer's consent to become entitled to any payment from the buyer. In his well-known treatise on law and economics, Posner (2003) describes the reluctance of the law to follow a Restitution Rule as follows:

> *…now suppose that a man stands under my window, playing the violin beautifully, and when he has finished knocks on my door and demands a fee for his efforts. Though I enjoyed the playing I refuse to pay anything for it. The court would deny the violinist's claim for a fee -- however reasonable the fee might appear to be – on the ground that, although the violinist conferred a benefit on me (and not with the intent that it be gratuitous), he did so officiously. Translated from legal into economic terminology, this means he*

> *conferred an unbargained-for benefit in circumstances where the costs of a voluntary bargain would have been low. In such cases the law insists that the voluntary route be followed – and is on firm economic grounds in doing so."* (p. 136)

We will focus in this paper on the use of the Mutual Consent Rule for situations in which bargaining is easy. As Posner states in the paragraph quoted above, those are the situations in which the case for using the Mutual Consent Rule appears to be compelling. However, while we will argue that this case is indeed strong, the case cannot be based on the conventional explanations given to the rule.

The standard explanation is that when the parties can easily bargain we would like to force buyers and sellers to get the other side's consent to the exchange. The requirement of mutual consent is viewed as an instrument of ensuring that an exchange would take place if and only if it is efficient. Courts are bound to make mistakes and estimate imperfectly values to the buyer and seller. Only the buyer and the seller are in a position to know the value of an exchange to them. When an exchange is efficient they will find a way to carry it out so that both will benefit. And when an exchange is inefficient, requiring mutual consent would guarantee its prevention since there will be no way to carry it out so that both sides will benefit.

While this explanation appears plausible at first glance, it is hardly persuasive upon closer look. The Restitution Rule does not prevent parties from bargaining. Even when the violinist in Posner's example is backed by a Restitution Rule, he might well bargain with the Posner before any playing takes place, if bargaining could produce a surplus. The presence of the Restitution Rule would just provide a different background for the bargaining – the bargaining will be conducted against a background in which, if no agreement is reached, the violinist will have a put option with an exercise price equal not to zero (as under the Mutual Consent Rule) but of the court-estimated value of the playing to Posner.

In situations in which bargaining is easy, supporters of the Mutual Consent Rule believe, this rule would generally result in an exchange taking place if and only if it is efficient. Although this might be true, a Restitution Rule would also lead to an efficient outcome in this setting. If the court is expected to under-estimate or over-estimate the value of the playing to Posner in a way that would produce an inefficient outcome absent bargaining, an inefficient outcome would be prevented because the parties would bargain around it. The conventional ex post argument favoring the Mutual Consent Rule over the Restitution Rule does not hold-up to scrutiny. In this paper we confirm the conventional wisdom supporting the Mutual Consent Rule. We argue, however, that the advantage of the Mutual Consent Rule is in ensuring ex ante rather than ex post efficiency.

The model to be developed in section 2 is one of a market in which potential buyers and sellers enter the market and then might invest in searching for (or avoiding) potential partners and in increasing opportunities for an exchange. If a buyer and a seller meet, we assume that there will be no impediments to bargaining so that the ex post outcome would always be efficient regardless of the legal regime. However, while the outcome would always be ex post efficient, the payoffs to the parties would differ from rule to rule, and as a result the parties' ex ante behavior would differ from rule to rule. Specifically, we show that sellers systematically benefit and buyers are systematically hurt from a Restitution Rule, as compared to the Mutual Consent Rule. As a result, sellers will invest excessively in searching for potential buyers and buyers will inefficiently invest in hiding from sellers.

Under the Restitution Rule, the parties' payoffs and resulting ex ante behavior depends on the court's estimate of the buyer's valuation. We assume that the court's estimate, which defines the exercise price of the seller's put option, is imperfect but unbiased. That is, we assume that courts make zero-mean random errors. At first glance, it seems that a party whose value is estimated by a court would not be hurt from the presence of such unbiased errors, but that does not turn out to be the case. To see this, reconsider the violin playing example and suppose that the court is equally likely to under- and over-estimate the value of the violin playing to Posner.

Consider first the case of an efficient exchange, in which the cost of playing to the violinist is 150 and the value to Posner is 200. Assume that the court has a 50-50 chance of estimating the value to Posner at 100 (under-estimation) and at 300 (over-estimation). Further assume that the violinist and Posner will recognize the direction of the court's error only once the violinist comes (when both sides see each other). In this case, Posner would be hurt by the prospect of over-valuation more than he would benefit form the prospect of under-valuation, and the violinist would correspondingly benefit more from the prospect of over-valuation than he would be hurt by the prospect of under-valuation. If once the violinist comes it turns out that an over-valuation at 300 is to be expected, the violinist will capture it fully. Posner will have to pay 300 and lose 100, and the violinist will make 100 beyond the surplus of 50 created. In contrast, if an under-valuation at 100 is expected, Posner will not be able to pocket the full magnitude of the under-valuation. The reason is that the under-valuation is not in Posner's pocket – Posner would capture it only if the violinist chooses to exercise his option, and the violinist thus will still have, under the Restitution Rule, a veto power on Posner's getting a gain from the under-valuation. In the considered example, since the violinist will not be expected to exercise the option in the event that the bargaining fails, he and Posner will bargain within the 150-200 range, and Posner will capture a gain of less than 50.

Consider now the possibility that the exchange would be inefficient because the cost to the violinist is 250 whereas the benefit to Posner remains 200. As before, assume the court has a 50-50 chance of estimating the value to Posner at 100 (under-estimation) and at 300 (over-estimation). Under the Mutual Consent Rule, neither party would have an incentive to look for the other or to hide from the other. The inefficient exchange will not occur and, more importantly for present purposes, will not lead to any socially wasteful investment. But under the Restitution Rule this would not be the case. Both parties will recognize that should the violinist come by when Posner is in the house the violinist would make an expected gain and Posner would make an expected loss. If upon seeing each other the parties recognize that the court is expected to over-value at 300, then the violinist would be able to extract from Posner a payment in return for not playing. If upon seeing each other the parties recognize that the court is expected to under-value at 100 the violinist would simply leave without profit. Thus, even when an exchange would be inefficient, under the Restitution Rule potential sellers would have an incentive to look for opportunities to make it, and potential buyers would have an incentive to incur costs to avoid such opportunities.

It follows from the above example that any visit by a violinist to Posner's home will result in Posner making an expected loss and the violinist making an expected gain larger than the expected social surplus. It follows that not only will Posner not have any incentive to invest in enhancing the likelihood of an exchange, he would even have an incentive to incur costs to reduce the likelihood of a meeting. Thus, Posner might have an incentive not to come home even if he would otherwise prefer to be home. And the violinist would have a socially excessive incentive to seek a meeting, spending too much time under Posner's window waiting for him to come home.

How bad would this be? It could be very bad if there are many violinists who could come by or, alternatively, if the same violinist could come many times. In this case, the expected loss to Posner might be large enough to impoverish him. For example, if upon seeing each other the parties recognize that the court will over-estimate the value of the playing to Posner, and if the violinist could play each day, and if Posner cannot hide, the violinist will be able to extract from Posner for a promise not to come at all a very large amount. And then another violinist can show up…. To use an example closer to most readers' lives, under a Restitution Rule with imperfect but unbiased court estimates, each of us who can be found by sellers of disks and books by mail will face the threat of becoming destitute. The above implies that, in the face of a Restitution Rule, potential buyers would have to completely hide in the sense of withdrawing from the market. If Massachusetts were to adopt a Restitution Rule, Bebchuk might have no choice but to join Bar-Gill in New York, a jurisdiction not using a Restitution Rule, before an army of sellers of disks and books start knocking on the doors of his home and office.

These detrimental implications of the Restitution Rule derive from the assumed information structure. Specifically, we assume that the seller and buyer, upon meeting each other, learn the direction of the court's error. The premise is that the parties obtain some information that is unavailable to the court. Or, put differently, we assume that some information is observable but not verifiable. While the assumption that information can be observable but not verifiable is standard in the contract theory literature, it merits some justification in the present context. Specifically, why would sellers have better information than courts in anonymous markets? First, we acknowledge that much information that sellers learn is (or can be made) verifiable to a court. All we need is that some, perhaps small, amount of information is difficult to convey to a court. Second, with current technology and marketing strategies anonymous markets are not quite so anonymous. Sellers invest heavily in acquiring information – from demographics to information about consumption patterns – about potential buyers. More importantly, the information structure is endogenous to the legal regime. And under the Restitution Rule sellers will have a strong incentive to acquire information that is not verifiable to courts.

Saul Levmore put forward a related argument against the restitution regime (Levmore 1985). Like us, Levmore argues that a Restitution Rule would impede the workings of the market. Under such a rule, Levmore observes, sellers will have an incentive to unilaterally confer benefits on buyers rather than compete for buyers' business in the market. And this is welfare reducing because, absent market-based competition, there is no guarantee that a buyer will be served by the seller who is best-suited for the job, i.e., can perform at the lowest cost. Our analysis reveals another distortion in sellers' incentives under the Restitution Rule. More importantly, we demonstrate that the Restitution Rule can significantly distort buyers' incentives—to the point that the demand side of the market will completely collapse.

This paper focuses on the Restitution Rule, which stands as a real-world alternative to the Mutual Consent Rule, at least under certain conditions. The Restitution Rule, however, is only one example of a pricing rule, i.e., a rule that gives the seller a put option to force the sale of a good or service at a court-determined price. Under the Restitution Rule the option's exercise price equals the benefit to the buyer. But rules setting different exercise prices can be easily imagined. A pricing rule with a court-determined price above the buyer's valuation produces the same adverse results as the Restitution Rule. Specifically, potential buyers might completely withdraw from the market. Such harmful results may not arise under a pricing rule with a court-determined price that is sufficiently below the buyer's valuation.

While we focus on the Restitution Rule, our analysis applies also to the symmetric "Takings" Rule. Under this rule, a potential buyer would be entitled to take an asset from a seller and become thereby only required to pay, absent an agreement between the parties to indicate otherwise, an amount equal to the value of the asset to the seller. (Using the influential taxonomy proposed by Calabresi and Melamed 1972, the Takings Rule provides liability rule, rather than property right, protection to the seller.) In the case of a new asset, the interpretation of the Takings Rule would imply giving a potential buyer the right to elect to have the seller produce the asset in return for the seller's cost of providing the asset as estimated by a court. That is, under this rule, potential buyers have a "call" option giving them the right to get an asset for an exercise price equal to the cost to the potential seller of giving up the asset or producing it (as the case might be). In contrast, under the Mutual Consent Rule, potential buyers can be viewed as having a (worthless) call option with an exercise price of infinity.[1]

Under the Restitution Rule "sellers" have an excessive incentive to seek out "buyers," and "buyers" have an incentive to hide from "sellers." The opposite is true under the Takings Rule: "buyers" have an excessive incentive to seek out "sellers," and "sellers" have an incentive to hide from "buyers." These adverse incentive effects of the Takings Rule have been identified in an important article by Louis Kaplow and Steve Shavell (1996). We formalize Kaplow and Shavell's observations as to the ex ante costs of the Takings Rule. Our main contributions, however, are (1) in identifying and analyzing the ex ante costs of the Restitution Rule (which is not studied in Kaplow and Shavell 1996),[2] and (2) in providing a general framework for comparing the Mutual Consent Rule with alternative pricing rules.

---

[1] The law does impose a Takings Rule in exceptional situations in which high (or infinite) transaction costs make the acquisition of consent impossible or impractical. Thus, because of holdout problems, government has an eminent domain power. And one has the right to moor one's boat at another's dock in a storm, even without the dock owner's consent, provided only that one pay afterwards the resulting costs to the dock's owner. However, these deviations from the Mutual Consent Rule are again exceptions. Generally, a potential buyer may get a benefit from a potential seller in exchange for a payment only upon obtaining the seller's consent to the exchange.

[2] As a terminological note, our Restitution Rule controls in situations where a seller forces a trade on a buyer. The literature has sometimes referred to a restitution *remedy* that a seller may be entitled to when a trade is forced by a buyer (as under our Takings Rule). The restitution remedy may include disgorgement of any profits that the buyer made as a result of the taking. For example, Schankerman and Scotchmer (2001) study the restitution remedy in the intellectual property context, adopting an ex ante approach similar to the one used in this paper.

Our analysis is close in its analytical approach to the work on incomplete contracting (see, e.g., Hart 1995). The incomplete contracting literature has analyzed how ex post divisions of value affect ex ante investments. This literature has focused, however, on contexts in which investments that affect ultimate payoffs are made after two parties make an incomplete contract that (due to problems of verifiability) does not specify these investments. In contrast, the context studied in this paper is one in which investments are made before parties meet and have a chance to make their initial contract. Note that, because in the studied context the investments are made before the parties meet, the analysis does not have to address the type of "foundations" problems that standard incomplete contracts models now need to deal with (see, e.g., Maskin and Tirole 1999 and Hart and Moore 1999).

The remainder of the paper is organized as follows. Section 2 describes the framework of analysis. Section 3 solves the model and demonstrates the ex ante cost of the Restitution Rule. Section 4 considers two extensions: general pricing rules and the Takings Rule. Section 5 offers concluding remarks.

2 Framework of Analysis

2.1 Sequence of Events

The model focuses on two economic actors (individuals or firms): a potential buyer, B, and a potential seller, S. Both B and S are assumed to be risk neutral with a discount rate of zero. The sequence of events in the model, which is illustrated in Figure 1 below, is as follows:

T = 0: B and S decide whether to enter the market.

T = 1: B and S do not know each other. However, they make *ex ante* investments, which affect (i) the likelihood that they will meet, and (ii) the potential surplus from a transfer if they meet.

T = 2: B and S meet with a certain probability, P. If they meet, S might transfer an existing asset, or produce and transfer a new asset, to B. In the event that a transfer takes place, it will increase total value by W (which might be positive or negative).

T = 3: Payoffs materialize.

2.2 Two Cases

The formulation used is sufficiently general to cover two cases. The term exchange will be used to indicate one of two cases – (i) S transfers an existing asset to

B in return for a payment, or (ii) S produces a new asset and transfers it to B in exchange for a payment. In both cases, if an exchange takes place, S gives up a value of C_S, where in the existing asset case, C_S denotes the use-value of the asset to S, and in the new asset case, C_S denotes the cost to S of producing the asset. Also, in both cases, if an exchange takes place, B obtains the asset. Let $V_B = C_S + W$ denote the value of the asset to B. As will be discussed, *W*, and thus whether an exchange would create a surplus, might depend on the state of nature.

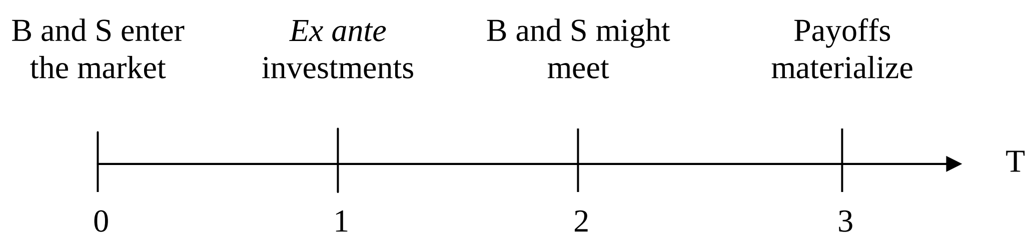

Fig. 1: The Sequence of Events

We shall now turn to specify the assumptions regarding each of the four stages.

2.3 T = 0: Entering the Market

At T = 0, B and S decide whether to enter the market. We solve the model under two alternative assumptions: (1) B and S can costlessly decide to enter (or exit) the market; and (2) B and S are already in the market at T = 0 and cannot exit. Clearly, if either party decides not to enter the market, the rest of the analysis becomes moot, and specifically any chance of a surplus-creating exchange is eliminated.

2.4 T = 1: Ex ante Investments

In either one of the two cases, the existing asset case and the new asset case, a prerequisite for a surplus-creating transaction is that the parties meet at T = 2. At T = 1, B and S can make investments, x_B and x_S, respectively, that increase the probability of a meeting at T = 2, *P*. Also at T = 1 the parties can make investments to avoid a meeting. Specifically, I assume that B and S can make investments, q_B and q_S, that lower the probability of a meeting. Formally, $P = P(x_B, x_S, q_B, q_S)$. We adopt the standard assumptions of diminishing returns to investments in increasing and decreasing *P*, i.e. $P'_{x_B}(x_B, x_S, q_B, q_S) > 0$, $P''_{x_B}(x_B, x_S, q_B, q_S) < 0$, $P'_{x_S}(x_B, x_S, q_B, q_S) > 0$,

$P''_{x_S}(x_B, x_S, q_B, q_S) < 0$, $P'_{q_B}(x_B, x_S, q_B, q_S) < 0$, $P''_{q_B}(x_B, x_S, q_B, q_S) > 0$, $P'_{q_S}(x_B, x_S, q_B, q_S) < 0$, and $P''_{q_S}(x_B, x_S, q_B, q_S) > 0$.

If the parties meet at T = 2, the surplus (if any) that would be created in the event of an exchange is *W*. At T = 1, B and S can make investments, y_B and y_S, respectively, that increase the value of the potential surplus *W*. Formally, we assume that $W = W(y_B, y_S) = G(y_B, y_S) + \mu$. The random variable μ, which represents the state of nature at T = 2, has a zero-mean distribution, characterized by the probability density function, $f(\cdot)$, and the cumulative distribution function, $F(\cdot)$. We will refer to the state nature as "good" if *W* is positive. With respect to $G(y_B, y_S)$, we adopt the standard assumptions of diminishing returns to investments in increasing *G*, i.e. $G'_{y_B}(y_B, y_S) > 0$, $G''_{y_B}(y_B, y_S) < 0$, $G'_{y_S}(y_B, y_S) > 0$, and $G''_{y_S}(y_B, y_S) < 0$. We assume that at T = 1, the parties do not know the state of nature μ.

Since B and S do not know each other at T = 1, when they make their *ex ante* investments, they cannot contract at T = 1, neither regarding their levels of investment, nor regarding the rules that will govern their future negotiations should they meet at T = 2.

2.5 T = 2: B and S Might Meet

At T = 2, B and S meet with probability *P*. If B and S meet, the following might occur. In the existing asset case, S might transfer the asset to B and get a payment π. In the new asset case, S might produce an asset, transfer it to B and get a payment π.

It is assumed that at T = 2, if B and S meet, they learn the realization of μ, the state of nature affecting *W*, and since *G* and the investment levels are assumed to be common knowledge, C_S and V_B are all common knowledge at this stage. The assumption that C_S and V_B are all common knowledge, together with the assumption we make that transaction costs are zero, ensure that the outcome will be ex post efficient under the two rules to be considered. That is, an exchange will take place *if and only if* *W* > 0 – that is, if and only if $V_B > C_S$. While the outcome will be ex post efficient under both rules, the rules will affect how the surplus (if any) will be divided and therefore in turn will affect ex ante investments and ex ante efficiency. Two different legal regimes will be considered:

(i) Mutual Consent Rule (MC): This is the familiar rule under which an asset will be transferred, or produced and transferred, in exchange for a payment by B, *if and only if* both parties agree for this to happen. To enforce this rule courts need to be able to verify only whether transfer and payments occur and whether mutual consent was given. We assume that courts have the requisite information.

Under the Mutual Consent rule, if the parties meet, and a positive surplus W exists, it will be assumed that S will make an expected gain of θW and B will make an expected gain of $(1-\theta)W$ with $\theta \in [0,1]$. This corresponds to the standard assumption that S can makes a take-it-or-leave-it offer with probability θ and B can make such an offer with a probability $1-\theta$.

(ii) Restitution Rule (R): Under the Restitution Rule, S may give the existing asset to B, or produce the asset and give it to B, and thereby become entitled (without B's consent being required) to the court-estimated value of V_B. Let E_B denote the court's estimate. We allow for court errors in assessing V_B. Specifically, the court's estimate of V_B is assumed to be $E_B = V_B + \varepsilon$. The error term, ε, is a random variable with a zero-mean distribution, characterized by the probability density function, $k(\cdot)$, and the cumulative distribution function, $K(\cdot)$.[3] We assume that before they decide how to act at T = 2, the parties know whether the court's estimate will be an over- or an under-valuation. (Otherwise, the imperfect information would have no effect on behavior given the assumption of risk-neutrality.)

With imperfect information, there will be sometimes reason for the parties to bargain. Having a "pricing" rule does not prohibit bargaining, it only gives one of the parties an option to act unilaterally. In the event that bargaining takes place, it will be assumed, as before, that S makes a take-it-or-leave-it offer with probability θ and B makes a take-it-or-leave-it-offer with probability $1-\theta$. The presence of the unilateral option, however, will affect what will happen if bargaining fails and thus shape the outcome. Specifically, when the party without the option makes the take-it-or-leave-it offer, the position of the party with the option is improved by the existence of the option. On the other hand, when the party with the option makes the take-it-or-leave-it offer the existence of the option may either improve or worsen the position of the party with the option, if that party cannot commit to give up the option. We will assume that such a commitment is impossible to make. The alternative assumption leads to results, which are qualitatively similar and quantitatively more supportive of the superiority of the Mutual Consent Rule. We shall comment on this alternative assumption in the footnotes.

In the event that an exchange takes place, let W_B denote B's gain (if any) from the exchange. Similarly, let W_S denote S's gain from the exchange. Of course, W_B and W_S must add up to W – the total social surplus (if any) from the exchange.

---

[3] The court's errors may be attributed to certain parameters of the state of nature, which are unobservable to the court.

2.6 T = 3: Final Payoffs

If an exchange took place at T = 2, then B will use the asset at T = 3, obtain V_B, and make a gain of $V_B - \pi$. Correspondingly, S will make a gain of $\pi - C_S$ from the exchange. If, on the other hand, an exchange does not take place at T = 2, then in the existing asset case S will use the asset at T = 3 and obtain C_S, and in the new asset case S will not produce the asset and thus save C_S. B does not receive an asset (and does not make any payment), and thus his payoff remains unchanged. For convenience, we normalize both parties' payoffs, where an exchange does not take place, to zero. Clearly, W, W_B and W_S will be all zero, if an exchange does not occur.

3 The Ex Ante Costs of the Restitution Rule

3.1 Optimal Entry and Investments

We begin by deriving the socially optimal entry and investment decisions. Since the expected surplus is non-negative (see below), it is optimal for both B and S to enter the market at T = 0. The optimal T = 1 investments can be derived from the expected social surplus function:

$$EW(x_B, x_S, q_B, q_S, y_B, y_S) = P \cdot \Pr[W > 0] \cdot E[W | W > 0] =$$

$$= P(x_B, x_S, q_B, q_S) \cdot \int_{-G(y_B, y_S)}^{\infty} [G(y_B, y_S) + \mu] f(\mu) d\mu$$

Lemma 1 characterizes the optimal entry and investment decisions.

Lemma 1: The optimal entry and investment decisions are as follows:
(i) Both B and S will enter the market.
(ii) B's optimal investment in increasing P, x_B^*, is characterized by –

$$(1) \quad \Pr[W > 0] \cdot E[W | W > 0] \cdot P'_{x_B}(x_B, x_S, q_B, q_S) = 1$$

B's optimal investment in reducing P, q_B^*, is zero.
(iii) S's optimal investment in increasing P, x_S^*, is characterized by –

$$(2) \quad \Pr[W > 0] \cdot E[W | W > 0] \cdot P'_{x_S}(x_B, x_S, q_B, q_S) = 1$$

S's optimal investment in reducing P, q_S^*, is zero.

(iv) B's optimal investment in increasing G, y_B^*, is characterized by –

$$(3)\quad P(x_B, x_S, q_B, q_S) \cdot [1 - F(-G(y_B, y_S))] \cdot G'_{y_B}(y_B, y_S) = 1$$

(v) S's optimal investment in increasing G, y_S^*, is characterized by –

$$(4)\quad P(x_B, x_S, q_B, q_S) \cdot [1 - F(-G(y_B, y_S))] \cdot G'_{y_S}(y_B, y_S) = 1$$

Remarks: The intuition for this result, whose detailed proof is omitted, is as follows:

(i) When the T = 1 investments are optimally set to maximize the expected social surplus (as explained below), market trade is welfare enhancing. Accordingly, it is optimal for both parties to enter the market. At T = 0.

(ii) It is socially desirable for B to invest in increasing the probability of a meeting as long as the added marginal surplus from such an increase, $\Pr[W > 0] \cdot E[W | W > 0] \cdot P'_{x_B}(x_B, x_S, q_B, q_S)$, exceeds the marginal cost of 1. From a social perspective there is no reason to invest in reducing the probability of a meeting.

(iii) The intuition for part (iii) of the lemma is similar to the intuition described for part (ii) of the lemma.

(iv) It is socially desirable for B to invest in increasing its benefits from the use of the asset, given that a meeting occurs and the asset is transferred to B, as long as the added marginal surplus from such an increase, $P(x_B, x_S, q_B, q_S) \cdot [1 - F(-G(y_B, y_S))] \cdot G'_{y_B}(y_B, y_S)$, exceeds the marginal cost of 1. The multiplier $P(x_B, x_S, q_B, q_S) \cdot [1 - F(-G(y_B, y_S))]$ reflects the fact that S and B will meet only with a certain probability and that given such a meeting a socially desirable transfer will occur only with a certain probability.

(v) The intuition for part (v) of the lemma is similar to the intuition described for part (iv) of the lemma.

3.2 The Mutual Consent Rule

Under the Mutual Consent Rule, if the parties meet, and if the state of nature is good (i.e., there is a positive surplus to be made from a transfer), an exchange will take place, and B will pay S a price of $\pi = (1-\theta) \cdot C_S + \theta \cdot (C_S + W) = C_S + \theta \cdot W$. As a result, the division of the surplus between B and S will be as follows:

$$W_B = V_B - \pi = V_B - (C_S + \theta \cdot W) = (1-\theta) \cdot W$$

$$W_S = \pi - C_S = \theta \cdot W$$

The entry and investment decisions under the Mutual Consent Rule, relative to the optimal entry and investment decisions, are as stated in the following proposition.

<u>Proposition 1</u>: The entry and investment decisions under the Mutual Consent Rule, as compared to the optimal entry and investment decisions, are –

(i) Both B and S will enter the market.

(ii) B's investment in increasing P, x_B^{MC}, will be sub-optimal, i.e., $x_B^{MC} < x_B^*$, and characterized by:

$$(5)\quad \Pr[W>0]\cdot(1-\theta)\cdot E[W|W>0]\cdot P'_{x_B}(x_B,x_S,q_B,q_S)=1$$

B's investment in reducing P will be optimal: $q_B^{MC} = q_B^* = 0$.

(iii) S's investment in increasing P, x_S^{MC}, will be sub-optimal, i.e., $x_S^{MC} < x_S^*$, and characterized by:

$$(6)\quad \Pr[W>0]\cdot\theta\cdot E[W|W>0]\cdot P'_{x_S}(x_B,x_S,q_B,q_S)=1$$

S's investment in reducing P will be optimal: $q_S^{MC} = q_S^* = 0$.

(iv) B's investment in increasing V_B, y_B^{MC}, will be sub-optimal, i.e., $y_B^{MC} < y_B^*$, and characterized by:

$$(7)\quad P(x_B,x_S,q_B,q_S)\cdot(1-\theta)\cdot[1-F(-G(y_B,y_S))]\cdot G'_{y_B}(y_B,y_S)=1$$

(v) S's investment in increasing V_S, y_S^{MC}, will be sub-optimal, i.e. $y_S^{MC} < y_S^*$, and characterized by:

$$(8)\quad P(x_B,x_S,q_B,q_S)\cdot\theta\cdot[1-F(-G(y_B,y_S))]\cdot G'_{y_S}(y_B,y_S)=1$$

Remarks: The intuition for this result, whose detailed proof is omitted, is as follows:

Under the Mutual Consent Rule each party chooses T = 1 investments to maximize her (non-negative) share of the expected social surplus (as explained below). Therefore, both parties expect to gain from market trade and thus choose to enter the market at T = 0.

For each one of the four investment options, x_B, x_S, y_B, and y_S, the party making the investment will capture only a fraction (θ in the case of S and $1-\theta$ in the case of B) of the social value produced by its investment. Therefore, for each one of the four investment options the level of investment will be sub-optimal. Neither party has a reason to avoid a meeting under the Mutual Consent Rule. Accordingly, investments to reduce the probability of a meeting, q_B and q_S, will be set at zero, which is optimal. Note that under the Mutual Consent Rule, judicial estimates of value are not used and thus potential errors in them do not matter.

3.3 The Restitution Rule

We derive the entry and investment decisions under the Restitution Rule under two alternative assumptions: (1) the parties do not anticipate the court's error at T = 1, and (2) the parties anticipate the court's error at T = 1.

3.3.1 The Parties Do Not Anticipate the Court's Error at T = 1

When the parties do not anticipate the court's error at T = 1 the entry and investment decisions under the Restitution Rule, relative to the optimal entry and investment decisions, are as stated in the following proposition.

Proposition 2: When the parties do not anticipate the court's error at T = 1, the entry and investment decisions under the Restitution Rule, as compared to the optimal entry and investment decisions, are –

(i) If possible, B will exit the market at T = 0.

(ii) If B cannot exit the market at T = 0, then -

- (a) B's investments, both in increasing *P*, x_B^R, and in increasing *W*, y_B^R, will be zero, i.e., $x_B^R = 0$ and $y_B^R = 0$. B will make a positive investment in reducing *P*, i.e., $q_B^R > q_B^* = 0$.
- (b) S will invest excessively both in increasing *P* and in increasing *W*, i.e., $x_S^R > x_S^*(x_B^R)$ and $y_S^R > y_S^*(y_B^R)$. (S will not invest in reducing *P*, i.e., $q_S^R = q_S^* = 0$.)

Proof:

When courts are imperfectly informed, $E[W_B] < 0$ and $E[W_S] > E[W]$. To see this examine the two following complementary scenarios, under the contingency that the parties meet at T = 2:

Scenario 1: $W > 0$ - An exchange takes place.

If $\varepsilon > 0$, i.e., the court over-estimates V_B, then S can unilaterally give the asset to B, such that B will lose ε, and S will gain the entire surplus plus ε. This determines the outcome of the bargaining process, namely B pays S a price off $\pi = V_B + \varepsilon$, and thus S gains $W_S = \pi - C_S = W + \varepsilon$ and B gains $W_B = V_B - \pi = -\varepsilon$. If $\varepsilon < 0$, i.e., the court under-estimates V_B, then – (a) if $-W < \varepsilon < 0$, then again S's background option to unilaterally give B the asset determines the outcome of the bargaining process, namely B pays S a price of $\pi = V_B + \varepsilon$, and thus S gains $W_S = \pi - C_S = W + \varepsilon > 0$ and B gains $W_B = V_B - \pi = -\varepsilon > 0$; and (b) if $\varepsilon < -W$, S's right to unilaterally give B the asset is moot, and the surplus from an exchange will be divided according to the parties' respective bargaining power, i.e., B pays S a price of $\pi = C_S + \theta \cdot W$, and thus S gains $W_S = \pi - C_S = \theta \cdot W$ and B gains $W_B = V_B - \pi = (1-\theta) \cdot W$.[4]

Scenario 2: $W < 0$ - An exchange does not take place.

If $\varepsilon < 0$, i.e., the court under-estimates V_B, then S will not exercise the right to unilaterally give B the asset, no bargaining will take place, and consequently both parties will remain with zero payoffs. If $\varepsilon > 0$, i.e., the court over-estimates V_B, then – (a) if $0 < \varepsilon < -W$, S will not exercise the right to unilaterally give B the asset, no bargaining will take place, and consequently both parties will remain with zero payoffs; (b) if $\varepsilon > -W$, S will extract from B an amount of $(1-\theta) \cdot W + \varepsilon$ in exchange for not exercising the unilateral right to give B the asset (the minimum S is willing to accept, i.e., S's reservation price, is: $E_B - C_S = W + \varepsilon$, and the maximum B is willing to pay, i.e., B's reservation price, is: $E_B - V_B = \varepsilon$), and thus S gains $W_S = \pi = (1-\theta) \cdot W + \varepsilon$ and B gains $W_B = -\pi = -[(1-\theta) \cdot W + \varepsilon]$.

The expected gain to B is:

$$
\begin{aligned}
E[W_B | W > 0] &= P \cdot \{\Pr[\varepsilon > 0] \cdot E[-\varepsilon | \varepsilon > 0] + \Pr[-W < \varepsilon < 0] \cdot E[-\varepsilon | -W < \varepsilon < 0] + \\
&\qquad + \Pr[\varepsilon < -W] \cdot E[(1-\theta) \cdot W | \varepsilon < -W]\} = \\
&= P \cdot \{-E[\varepsilon] + \Pr[\varepsilon < -W] \cdot E[(1-\theta) \cdot W + \varepsilon | \varepsilon < -W]\} = \\
&= P \cdot \{0 + \Pr[\varepsilon < -W] \cdot E[(1-\theta) \cdot W + \varepsilon | \varepsilon < -W]\} < 0
\end{aligned}
$$

$$
\begin{aligned}
E[W_B | W < 0] &= P \cdot \{\Pr[\varepsilon < 0] \cdot 0 + \Pr[0 < \varepsilon < -W] \cdot 0 + \Pr[\varepsilon > -W] \cdot E[-((1-\theta) \cdot W + \varepsilon) | \varepsilon > -W]\} = \\
&= P \cdot \{\Pr[\varepsilon < -W] \cdot 0 - \Pr[\varepsilon > -W] \cdot E[(1-\theta) \cdot W + \varepsilon | \varepsilon > -W]\} < 0
\end{aligned}
$$

Since the expected gain to B from a meeting is negative, B will exit the market at T = 0, if exit is possible. If exit is impossible, B will invest to lower *P*. B will not invest in increasing *W*.

The expected gain to S is, therefore:

[4] Under the alternative assumption that S can commit not to exercise the right to unilaterally give B the asset, S will make such a commitment if $-W < \varepsilon < -(1-\theta) \cdot W$. Therefore, for these levels of court error, S will gain more, compared to the no commitment assumption, i.e., $W_S^{\text{Commitment}} = \theta \cdot W > W + \varepsilon = W_S^{\text{No Commitment}}$, and correspondingly B will gain less, compared to the no commitment assumption.

$$E[W_S|W>0] = P\cdot\{\Pr[\varepsilon>0]\cdot E[W+\varepsilon|\varepsilon>0]+\Pr[-W<\varepsilon<0]\cdot E[W+\varepsilon|-W<\varepsilon<0]+$$
$$+\Pr[\varepsilon<-W]\cdot E[\theta\cdot W|\varepsilon<-W]\}=$$
$$= P\cdot\{W+E[\varepsilon]+\Pr[\varepsilon<-W]\cdot E[-(1-\theta)\cdot W-\varepsilon|\varepsilon<-W]\}=$$
$$= P\cdot\{W-\Pr[\varepsilon<-W]\cdot E[(1-\theta)\cdot W+\varepsilon|\varepsilon<-W]\}>P\cdot W$$

$$E[W_S|W<0] = P\cdot\{\Pr[\varepsilon<0]\cdot 0+\Pr[0<\varepsilon<-W]\cdot 0+\Pr[\varepsilon>-W]\cdot E[\varepsilon+(1-\theta)\cdot W|\varepsilon>-W]\}=$$
$$= P\cdot\{\Pr[\varepsilon<-W]\cdot 0+\Pr[\varepsilon>-W]\cdot E[\varepsilon+(1-\theta)\cdot W|\varepsilon>-W]\}>0$$

Since the expected gain to S from a meeting exceeds the expected social surplus, S will invest excessively in increasing *P*. S will also invest excessively in increasing *W*. QED

Remarks: The intuition for this result is as follows:

When courts are imperfectly informed, a meeting of the parties will result in $E[W_B]<0$ and $E[W_S]>E[W]$, i.e., B expects to lose from a meeting and S expects to gains from a meeting more than the total surplus. To see why B loses and S gains so much from a meeting, consider the two following complementary scenarios, under the contingency that the parties meet at T = 2:

Scenario 1: $W>0$ -- In this scenario, an exchange will produce a surplus and therefore will take place. In this scenario, S will fully benefit from possible over-valuation but will be only partly hurt by a possible under-valuation. If $\varepsilon>0$, i.e., the court over-estimates V_B, then S will make an extra gain of ε from the transfer and B will bear a loss of ε. However, when $\varepsilon<0$, i.e., when the court under-estimates V_B, B will not always make a gain of ε. The reason is that, because S does not have to give B the asset, S will not always have to "pay" the full ε. Specifically, when $\varepsilon<-W$, such that clearly S will not exercise the right to unilaterally give B the asset, the surplus *W* will be divided according to the parties respective bargaining power, and B will thus make a gain of less than ε.[5]

Scenario 2: $W<0$ - In this scenario, an exchange will destroy value and will not take place. In this scenario, if $\varepsilon<0$, i.e., the court under-estimates V_B, then S will not give B the asset, no bargaining will take place, and consequently S will not lose, and B will not gain, from the under-valuation. The same outcome obtains when the court over-estimates V_B, but the error is not too large, i.e., $0<\varepsilon<W$. However, if $\varepsilon>W$, B will have to bribe S in order to prevent S from exercising the right to unilaterally

[5] Under the alternative assumption that S can commit not to exercise the right to unilaterally give B the asset, S will make such a commitment if $-W<\varepsilon<-(1-\theta)\cdot W$, and B's gain will be even lower. See proof, *infra*.

give B the asset. In this case, then, B will lose and S will gain form the over-valuation.

The above analysis indicates that, if the parties meet, B will make an expected loss. Accordingly, if possible, B will exit the market at T = 0. If B cannot exit the market at T = 0, then at T = 1 B will invest in reducing the probability of a meeting (i.e., in "hiding" from S). As B's loss is S's gain, S will make in the event of a meeting an expected gain that is larger than the expected surplus. Therefore, S will invest excessively in increasing the probability of a meeting. Also, since S expects to obtain more than 100% of the surplus, S will invest excessively in increasing the surplus, *W*. Conversely, since B expects a net loss equal to a share of the surplus, B will not invest in increasing the surplus, *W*.

3.3.2 The Parties Anticipate the Court's Error at T = 1

When the parties anticipate the court's error at T=1 the entry and investment decisions under the Restitution Rule, relative to the optimal entry and investment decisions, are as stated in the following proposition.

<u>Proposition 3</u>: When the parties anticipate the court's error at T = 1, the entry and investment decisions under the Restitution Rule, as compared to the optimal entry and investment decisions, are –

(i) If possible, B will exit the market at T = 0.

(ii) If B cannot exit the market at T = 0, then –

(a) When the court is expected to err against B, i.e., to overestimate V_B $(\varepsilon > 0)$,

(1) B's investments, both in increasing *P*, x_B^R, and in increasing *W*, y_B^R, will be zero, i.e., $x_B^R = 0$ and $y_B^R = 0$. B will make a positive investment in reducing *P*, i.e., $q_B^R > q_B^* = 0$. Moreover, B's excessive investment will exceed the excessive investment characterized in Proposition 2.

(2) S will invest excessively both in increasing *P* and in increasing *W*, i.e., $x_S^R > x_S^*(x_B^R)$ and $y_S^R > y_S^*(y_B^R)$. Moreover, S's excessive investments will exceed the excessive investments characterized in Proposition 2. (S will not invest in reducing *P*, i.e., $q_S^R = q_S^* = 0$.)

(b) When the court is expected to err in B's favor, i.e., to underestimate V_B $(\varepsilon < 0)$, both B and S will invest (sub-optimally) in increasing *P* (neither will invest in lowering *P*). B's incentives to invest will be weaker and S's incentives to invest will be stronger, than under the Mutual Consent Rule.

Proof:

First, note that $E[W_B|W>0]$ and $E[W_B|W<0]$ are still

$$E[W_B|W>0]=P\cdot\{0+\Pr[\varepsilon<-W]\cdot E[(1-\theta)\cdot W+\varepsilon|\varepsilon<-W]\}<0$$ and

$$E[W_B|W<0]=P\cdot\{\Pr[\varepsilon<-W]\cdot 0-\Pr[\varepsilon>-W]\cdot E[(1-\theta)\cdot W+\varepsilon|\varepsilon>-W]\}<0,$$

as derived in the proof of proposition 2 (with different investment levels as described below). Since the expected gain to B from a meeting is negative, if possible B will exit the market at T=0.

Next, building upon the proof of proposition 2, we obtain:

When the court overestimates V_B, i.e. $\varepsilon>0$ -

$$E[W_S|(W>0)\cap(\varepsilon>0)]=P\cdot E[W+\varepsilon|\varepsilon>0]$$

$$E[W_S|(W<0)\cap(\varepsilon>0)]=P\cdot\{\Pr[0<\varepsilon<-W|\varepsilon>0]\cdot 0+\Pr[\varepsilon>-W|\varepsilon>0]\cdot E[\varepsilon+(1-\theta)\cdot W|\varepsilon>-W]\}$$

$$E[W_B|(W>0)\cap(\varepsilon>0)]=P\cdot E[-\varepsilon|\varepsilon>0]$$

$$E[W_B|(W<0)\cap(\varepsilon>0)]=P\cdot\{\Pr[0<\varepsilon<-W|\varepsilon>0]\cdot 0+\Pr[\varepsilon>-W|\varepsilon>0]\cdot E[-((1-\theta)\cdot W+\varepsilon)|\varepsilon>-W]\}$$

The results stated in part (ii)(a)(1) of the proposition follow from a comparison of the above expected gains to the expected gains derived in the proof of proposition 2.

When the court underestimates V_B, i.e. $\varepsilon<0$ -

$$E[W_S|(W>0)\cap(\varepsilon<0)]=P\cdot\{\Pr[-W<\varepsilon<0|\varepsilon<0]\cdot E[W+\varepsilon|-W<\varepsilon<0]+\Pr[\varepsilon<-W|\varepsilon<0]\cdot E[\theta\cdot W|\varepsilon<-W]\}=$$
$$=P\cdot\{\theta\cdot W+\Pr[-W<\varepsilon<0|\varepsilon<0]\cdot E[(1-\theta)\cdot W+\varepsilon|-W<\varepsilon<0]\}$$

$$E[W_S|(W<0)\cap(\varepsilon<0)]=P\cdot 0=0$$

$$E[W_B|(W>0)\cap(\varepsilon<0)]=P\cdot\{\Pr[-W<\varepsilon<0|\varepsilon<0]\cdot E[-\varepsilon|-W<\varepsilon<0]+\Pr[\varepsilon<-W|\varepsilon<0]\cdot E[(1-\theta)\cdot W|\varepsilon<-W]\}=$$
$$=P\cdot\{(1-\theta)\cdot W-\Pr[-W<\varepsilon<0|\varepsilon<0]\cdot E[(1-\theta)\cdot W+\varepsilon|-W<\varepsilon<0]\}$$

$$E[W_B|(W<0)\cap(\varepsilon<0)]=P\cdot 0=0$$

Since the expected gain to B from a meeting is positive, B will invest to increase P. Clearly, S will invest to increase P. To complete the proof of the results stated in part (ii)(a)(2) of the proposition the above expected gains need only be compared to the expected gains under the Mutual Consent Rule (recall that under the Mutual Consent Rule, the parties' expected gains are independent of the court's imperfect information): $E[W_S]=P\cdot\{\Pr[W>0]\cdot E[\theta\cdot W|W>0]\}$ and $E[W_B]=P\cdot\{\Pr[W>0]\cdot E[(1-\theta)\cdot W|W>0]\}$.

QED

Remarks: The intuition for this result is as follows:

As in proposition 2, from a T = 0 perspective B will make an expected loss. Accordingly, if possible, B will exit the market at T = 0.

When it is known that the court's error will be against B, i.e., the court will overestimate V_B ($\varepsilon>0$), the expected loss to B form a meeting would be larger as compared with the expected loss in proposition 2. The reason is that in proposition 2, there was a possibility that the court's error will be in favor of B and thus B would gain something from the exchange. The results stated in part (ii)(a)(i) of the proposition follow from this comparison.

When it is known that the court's error will be in B's favor, i.e., the court will underestimate V_B ($\varepsilon < 0$), B will be able to gain something from the exchange, but since S does not have to give B the asset, B will be in a worse position than under the Mutual Consent Rule. The situation is identical to the situation under the Mutual Consent Rule with a put option that S has to sell the asset at a somewhat favorable (to B) price – the option would make S better off and B worse off compared with the Mutual Consent Rule.

4 Extensions

4.1 Other Pricing Rules

The preceding analysis focused on the Restitution Rule and compared this rule to the prevailing Mutual Consent Rule. We chose to focus on the Restitution Rule because it stands as a real-world alternative to the Mutual Consent Rule, at least under certain conditions. The Restitution Rule, however, is only one example of a pricing rule, i.e., a rule that gives the seller a put option to force the sale of a good or service at a court-determined price. Under the Restitution Rule the option's exercise price equals the benefit to the buyer. But rules setting different exercise prices can be easily imagined.

Our model can be extended to study a generic pricing rule with a court-determined exercise price of π^R. The court still makes errors, such that the ex post price is $E_B = \pi^R + \varepsilon$. The entry and investment decisions under a general pricing rule, relative to the optimal entry and investment decisions, are as stated in the following proposition.

<u>Proposition 4</u>: Under a pricing rule with a court-determined exercise price of π^R (when the parties do not anticipate the court's error at T = 1), there exists a threshold value $\hat{\pi}^R < V_B$ such that –

(i) When $\pi^R > \hat{\pi}^R$ -

(a) If possible, B will exit the market at T = 0.

(b) If B cannot exit the market at T = 0, then -

(1) B's investments, both in increasing *P*, x_B^R, and in increasing *W*, y_B^R, will be zero, i.e., $x_B^R = 0$ and $y_B^R = 0$. B will make a positive investment in reducing *P*, i.e., $q_B^R > q_B^* = 0$.

(2) S will invest excessively both in increasing *P* and in increasing *W*, i.e., $x_S^R > x_S^*(x_B^R)$ and $y_S^R > y_S^*(y_B^R)$. (S will not invest in reducing *P*, i.e., $q_S^R = q_S^* = 0$.)

(ii) When $\pi^R < \hat{\pi}^R$,

(a) B and S will both enter the market at T = 0.

(b) B's investments, both in increasing *P*, x_B^R, and in increasing *W*, y_B^R, will be sub-optimal, i.e., $x_B^R < x_B^*$ and $y_B^R < y_B^*$. B's investment in reducing *P* will be optimal: $q_B^R = q_B^* = 0$.

(c) S's investments, both in increasing *P*, x_S^R, and in increasing *W*, y_S^R, will be sub-optimal, i.e., $x_S^R < x_S^*$ and $y_S^R < y_S^*$. S's investment in reducing *P* will be optimal: $q_S^R = q_S^* = 0$.

Proof:

As in proposition 2, the central question is whether the ex post price exceeds the buyer's valuation, V_B, i.e., whether $E_B > V_B$ or $\varepsilon > V_B - \pi^R$. Letting $\hat{\varepsilon} = V_B - \pi^R$, we can generalize the proof of proposition 2 as follows (note that under the Restitution Rule, where $\pi^R = V_B$, $\hat{\varepsilon} = 0$):

Examine the two complementary scenarios from proposition 2, under the contingency that the parties meet at T = 2:

Scenario 1: $W > 0$ - An exchange takes place.

If $\varepsilon > \hat{\varepsilon}$, then S can unilaterally give the asset to B. This determines the outcome of the bargaining process, namely B pays S a price of $\pi = \pi^R + \varepsilon$, and thus S gains $W_S = \pi - C_S = W + (\varepsilon - \hat{\varepsilon}) > W$ and B gains $W_B = V_B - \pi = -(\varepsilon - \hat{\varepsilon}) < 0$. If $\varepsilon < \hat{\varepsilon}$, then – (a) if $\hat{\varepsilon} - W < \varepsilon < \hat{\varepsilon}$, then again S's background option to unilaterally give B the asset determines the outcome of the bargaining process, namely B pays S a price of $\pi = \pi^R + \varepsilon$, and thus S gains $W_S = \pi - C_S = W + (\varepsilon - \hat{\varepsilon}) > 0$ and B gains $W_B = V_B - \pi = -(\varepsilon - \hat{\varepsilon}) > 0$; and (b) if $\varepsilon < \hat{\varepsilon} - W$, S's right to unilaterally give B the asset is moot, and the surplus from an exchange will be divided according to the parties' respective bargaining power, i.e. B pays S a price of $\pi = C_S + \theta \cdot W$, and thus S gains $W_S = \pi - C_S = \theta \cdot W$ and B gains $W_B = V_B - \pi = (1 - \theta) \cdot W$.[6]

[6] Under the alternative assumption that S can commit not to exercise the right to unilaterally give B the asset, S will make such a commitment if $\hat{\varepsilon} - W < \varepsilon < \hat{\varepsilon} - (1 - \theta) \cdot W$. Therefore, for these levels of court error, S will gain more, compared to the no commitment assumption, i.e.,

Scenario 2: $W < 0$ - An exchange does not take place.

If $\varepsilon < \hat{\varepsilon}$, then S will not exercise the right to unilaterally give B the asset, no bargaining will take place, and consequently both parties remain with zero payoffs. If $\varepsilon > \hat{\varepsilon}$, then – (a) if $\hat{\varepsilon} < \varepsilon < \hat{\varepsilon} - W$, S will not exercise the right to unilaterally give B the asset, no bargaining will take place, and consequently both parties remain with zero payoffs; (b) if $\varepsilon > \hat{\varepsilon} - W$, S will extract from B an amount of $(1-\theta)\cdot W + (\varepsilon - \hat{\varepsilon})$ in exchange for not exercising the unilateral right to give B the asset (the minimum S is willing to accept, i.e. S's reservation price, is: $E_B - C_S = W + (\varepsilon - \hat{\varepsilon})$, and the maximum B is willing to pay, i.e. B's reservation price, is: $E_B - V_B = \varepsilon - \hat{\varepsilon}$), and thus S gains $W_S = \pi = (1-\theta)\cdot W + (\varepsilon - \hat{\varepsilon})$ and B gains $W_B = -\pi = -[(1-\theta)\cdot W + (\varepsilon - \hat{\varepsilon})]$.

The expected gain to S is, therefore:

$$E[W_S|W > 0] = P\cdot\{\Pr[\varepsilon > \hat{\varepsilon}]\cdot E[W + (\varepsilon - \hat{\varepsilon})|\varepsilon > \hat{\varepsilon}] + \Pr[\hat{\varepsilon} - W < \varepsilon < \hat{\varepsilon}]\cdot E[W + (\varepsilon - \hat{\varepsilon})|\hat{\varepsilon} - W < \varepsilon < \hat{\varepsilon}] +$$
$$+ \Pr[\varepsilon < \hat{\varepsilon} - W]\cdot E[\theta\cdot W|\varepsilon < -W]\} =$$
$$= P\cdot\{W + E[\varepsilon - \hat{\varepsilon}] + \Pr[\varepsilon < \hat{\varepsilon} - W]\cdot E[-(1-\theta)\cdot W - (\varepsilon - \hat{\varepsilon})|\varepsilon < \hat{\varepsilon} - W]\} =$$
$$= P\cdot\{W - \hat{\varepsilon} - \Pr[\varepsilon < \hat{\varepsilon} - W]\cdot E[(1-\theta)\cdot W + (\varepsilon - \hat{\varepsilon})|\varepsilon < \hat{\varepsilon} - W]\}$$

$$E[W_S|W < 0] = P\cdot\{\Pr[\varepsilon < \hat{\varepsilon}]\cdot 0 + \Pr[\hat{\varepsilon} < \varepsilon < \hat{\varepsilon} - W]\cdot 0 + \Pr[\varepsilon > \hat{\varepsilon} - W]\cdot E[(\varepsilon - \hat{\varepsilon}) + (1-\theta)\cdot W|\varepsilon > \hat{\varepsilon} - W]\} =$$
$$= P\cdot\{\Pr[\varepsilon < \hat{\varepsilon} - W]\cdot 0 + \Pr[\varepsilon > \hat{\varepsilon} - W]\cdot E[(\varepsilon - \hat{\varepsilon}) + (1-\theta)\cdot W|\varepsilon > \hat{\varepsilon} - W]\} > 0$$

The expected gain to B is:

$$E[W_B|W > 0] = P\cdot\{\Pr[\varepsilon > \hat{\varepsilon}]\cdot E[-(\varepsilon - \hat{\varepsilon})|\varepsilon > \hat{\varepsilon}] + \Pr[\hat{\varepsilon} - W < \varepsilon < \hat{\varepsilon}]\cdot E[-(\varepsilon - \hat{\varepsilon})|\hat{\varepsilon} - W < \varepsilon < \hat{\varepsilon}] +$$
$$+ \Pr[\varepsilon < \hat{\varepsilon} - W]\cdot E[(1-\theta)\cdot W|\varepsilon < \hat{\varepsilon} - W]\} =$$
$$= P\cdot\{-E[\varepsilon - \hat{\varepsilon}] + \Pr[\varepsilon < \hat{\varepsilon} - W]\cdot E[(1-\theta)\cdot W + (\varepsilon - \hat{\varepsilon})|\varepsilon < \hat{\varepsilon} - W]\} =$$
$$= P\cdot\{\hat{\varepsilon} + \Pr[\varepsilon < \hat{\varepsilon} - W]\cdot E[(1-\theta)\cdot W + (\varepsilon - \hat{\varepsilon})|\varepsilon < \hat{\varepsilon} - W]\}$$

$$E[W_B|W < 0] = P\cdot\{\Pr[\varepsilon < \hat{\varepsilon}]\cdot 0 + \Pr[\hat{\varepsilon} < \varepsilon < \hat{\varepsilon} - W]\cdot 0 + \Pr[\varepsilon > \hat{\varepsilon} - W]\cdot E[-[(1-\theta)\cdot W + (\varepsilon - \hat{\varepsilon})]|\varepsilon > \hat{\varepsilon} - W]\} =$$
$$= P\cdot\{\Pr[\varepsilon < \hat{\varepsilon} - W]\cdot 0 - \Pr[\varepsilon > \hat{\varepsilon} - W]\cdot E[(1-\theta)\cdot W + (\varepsilon - \hat{\varepsilon})|\varepsilon > \hat{\varepsilon} - W]\} < 0$$

Since the conditional expected gain to S, $E[W_S|W > 0]$, can be smaller than $P\cdot W$, and correspondingly, the conditional expected gain to B, $E[W_B|W > 0]$, can be greater than zero, the $E[W_B] < 0$ and $E[W_S] > E[W]$ result may not hold. Moreover, with continuity, there exists a threshold value $\hat{\pi}^R < V_B$ such that $E[W_B] < 0$ and $E[W_S] > E[W]$ if and only if $\pi^R > \hat{\pi}^R$.

QED

Remarks: The intuition for this result is as follows:

$W_S^{\text{Commitment}} = \theta\cdot W > W + (\varepsilon - \hat{\varepsilon}) = W_S^{\text{No Commitment}}$, and correspondingly B will gain less, compared to the no commitment assumption.

When $\pi^R > V_B$, the results are qualitatively similar to those obtained under the Restitution Rule (see proposition 2) and for the same reasons.

When $\pi^R < V_B$, however, it is no longer necessarily the case that $E[W_B] < 0$ and $E[W_S] > E[W]$, and, therefore, the adverse ex ante implications of the Restitution Rule need not generalize to pricing rules with $\pi^R < V_B$. As explained in the remarks to proposition 2, the $E[W_B] < 0$ and $E[W_S] > E[W]$ result under the Restitution Rule follows from the observation that S will fully benefit from possible over-valuation but will be only partly hurt by a possible under-valuation. Together with the observation that S gains *more than W* in the over-valuation case and the assumption that court errors are symmetrically distributed around zero, this produces the $E[W_B] < 0$ and $E[W_S] > E[W]$ result. When $\pi^R < V_B$, $E_B > V_B$, the equivalent of over-valuation under the Restitution Rule, requires $\varepsilon > V_B - \pi^R$. It is less likely that S will gain more than W, and thus we may no longer get the $E[W_B] < 0$ and $E[W_S] > E[W]$ result. Moreover, with continuity, there exists a threshold value $\hat{\pi}^R < V_B$ such that $E[W_B] < 0$ and $E[W_S] > E[W]$ if and only if $\pi^R > \hat{\pi}^R$.

4.2 The Takings Rule

We demonstrated the disadvantage of the Restitution Rule as compared to the Mutual Consent Rule. Recognizing that the Restitution Rule is the mirror image of the well-studied Takings Rule, our analysis can be readily extended to the Takings Rule. The application of our analysis to the Takings Rule formalizes the arguments made by Kaplow and Shavell (1996).

We begin by restating the Takings Rule using our notation.

Takings Rule (T): Under the Takings Rule, B may take the existing asset and use it, or get an injunction requiring S to produce the asset and then transfer it to B, provided only that B makes S whole by giving S the court-estimated value of C_S. Let E_S denote the court's estimate. We allow for court errors in assessing C_S. Specifically, the court's estimate of C_S is assumed to be $E_S = C_S + \gamma$. The error term, γ, is a random variable with a zero-mean distribution, characterized by the probability density function, $h(\cdot)$, and the cumulative distribution function, $H(\cdot)$. We assume that before they decide how to act at T = 2, the parties know whether the court's estimate will be an over- or an under-valuation. (Otherwise, the imperfect information would have no effect on behavior given the assumption of risk-neutrality.)

We derive the entry and investment decisions under the Takings Rule under two alternative assumptions: (1) the parties do not anticipate the court's error at T = 1, and (2) the parties anticipate the court's error at T = 1.

4.2.1 The Parties Do Not Anticipate the Court's Error at T = 1

When the parties do not anticipate the court's error at T = 1 the entry and investment decisions under the Takings Rule, relative to the optimal entry and investment decisions, are as stated in the following proposition.

Proposition 5: When the parties do not anticipate the court's error at T = 1, the entry and investment decisions under the Takings Rule, as compared to the optimal entry and investment decisions, are –

(i) If possible, S will exit the market at T = 0.

(ii) If S cannot exit the market at T = 0, then -

(a) S's investments, both in increasing *P*, x_S^T, and in increasing *W*, y_S^T, will be zero, i.e., $x_S^T = 0$ and $y_S^T = 0$. S will make a positive investment in reducing *P*, i.e., $q_S^T > q_S^* = 0$.

(b) B will invest excessively both in increasing *P* and in increasing *W*, i.e., $x_B^T > x_B^*(x_S^T)$ and $y_B^T > y_B^*(y_S^T)$. (B will not invest in reducing *P*, i.e., $q_B^T = q_B^* = 0$.)

Remarks: This result is the mirror image of the result stated in proposition 2 for the Restitution Rule. Therefore, the proof for this result is omitted. The intuition for this result is as follows:

When courts are imperfectly informed, a meeting of the parties will result in $E[W_S] < 0$ and $E[W_B] > E[W]$. To see why S loses from a meeting, consider the two following complementary scenarios, under the contingency that the parties meet at T = 2:

Scenario 1: $W > 0$ -- In this scenario, an exchange will produce a surplus and therefore will take place. In this scenario, B will fully benefit from possible under-valuation but will be only partly hurt by a possible over-valuation. If $\gamma < 0$, i.e., the court under-estimates C_S, then B will make an extra gain from the transfer of γ and S will bear such a loss. However, when $\gamma > 0$, i.e., when the court over-estimates C_S, S will not always make a gain of γ. The reason is that, because B does not have to take the asset, B will not always have to pay the full γ. Specifically, when $\gamma > W$, such that clearly B will not taking the asset, the surplus W will be divided according to the parties respective bargaining power, and S will thus make a gain of less than γ.[7]

Scenario 2: $W < 0$ - In this scenario, an exchange will destroy value and will not take place. In this scenario, if $\gamma > 0$, i.e., the court over-estimates C_S, then there will be no taking and no bargaining, and consequently B will not lose, and S will not gain, form the over-valuation. The same outcome obtains when the court under-estimates C_S, but the error is not too large, i.e. $W < \gamma < 0$. However, if $\gamma < W < 0$, S will have to bribe B in order to prevent B from taking the asset. In this case, then, S will lose and B will gain form the under-valuation.

The above analysis indicates that, if the parties meet, S will make an expected loss. It follows that not only will S not invest in searching for B, but also S will invest in reducing the probability of a meeting (i.e., in "hiding" from B). As S's loss is B's gain, B will make in the event of a meeting an expected gain that is larger than the expected surplus. Therefore, B will invest excessively in increasing the probability of a meeting.

Also, since B expects to obtain more than 100% of the surplus, B will invest excessively in increasing the surplus, W. Conversely, since S expects a net loss equal to a share of the surplus, S will not invest in increasing the surplus, W.

4.2.2 The Parties Anticipate the Court's Error at T = 1

When the parties anticipate the court's error at T=1 the entry and investment decisions under the Takings Rule, relative to the optimal entry and investment decisions, are as stated in the following proposition.

<u>Proposition 6</u>: When the parties anticipate the court's error at T = 1, the entry and investment decisions under the Takings Rule, as compared to the optimal entry and investment decisions, are –

(i) If possible, S will exit the market at T = 0.

(ii) If S cannot exit the market at T = 0, then –

- (a) When the court is expected to err against S, i.e., to underestimate C_S ($\gamma < 0$),
  - (1) S's investments, both in increasing P, x_S^T, and in increasing W, y_S^T, will be zero, i.e., $x_S^T = 0$ and $y_S^T = 0$. S will make a positive investment in reducing P, i.e., $q_S^T > q_S^* = 0$. Moreover, S's excessive investment will exceed the excessive investment characterized in Proposition 5.

[7] Under the alternative assumption that B can commit not to take the asset, B will make such a commitment if $\theta \cdot W < \gamma < W$, and S's gain will be even lower.

(2) B will invest excessively both in increasing *P* and in increasing *W*, i.e., $x_B^T > x_B^*(x_S^T)$ and $y_B^T > y_B^*(y_S^T)$. Moreover, B's excessive investments will exceed the excessive investments characterized in Proposition 5. (B will not invest in reducing *P*, i.e., $q_B^T = q_B^* = 0$.)

(b) When the court is expected to err in S's favor, i.e., to overestimate C_S $(\gamma > 0)$, both B and S will invest (sub-optimally) in increasing *P* (neither will invest in lowering *P*). S's incentives to invest will be weaker and B's incentives to invest will be stronger, than under the Mutual Consent Rule.

Remarks: This result is the mirror image of the result stated in proposition 3 for the Restitution Rule. Therefore, the proof for this result is omitted. The intuition for this result is as follows:

When it is known that the court's error will be against S, i.e., the court will underestimate V_S $(\gamma < 0)$, the expected loss to S form a meeting would be larger as compared with the expected loss in proposition 5. The reason is that in proposition 5, there was a possibility that the court's error will be in S's favor and thus S would gain something from the exchange. The results stated in part (i) of the proposition follow from this comparison.

When it is known that the court's error will be in S's favor, i.e., the court will overestimate V_S $(\gamma > 0)$, S will be able to gain something from the exchange, but since B does not have to take the asset, S will be in a worse position than under the Mutual Consent Rule. The situation is identical to the situation under the Mutual Consent Rule with a call option that B has to buy the asset at a somewhat favorable (to S) price – the option would make B better off and S worse off compared with the Mutual Consent Rule.

5 Concluding Remarks

[TO BE ADDED]

REFERENCES

Calabresi, Guido, and Douglas A. Melamed (1972), "Property Rules, Liability Rules and Inalienability: One View of the Cathedral," *Harvard Law Review*, 85, 1089-1128.

Hart, Oliver (1995), *Firms, Contracts, and Financial Structure* (Oxford University Press).

Hart, Oliver, and John Moore (1999), "Foundations of Incomplete Contracts," *Review of Economic Studies*, 66, 115-138.

Kaplow, Louis, and Steven Shavell (1996), "Property Rules versus Liability Rules: An Economic Analysis," *Harvard Law Review*, 109, 713-790.

Levmore, Saul (1985), "Explaining Restitution," *Virginia Law Review*, 71, 65-124.

Maskin, Eric, and Jean Tirole (1999), "Unforeseen Contingencies and Incomplete Contracts," *Review of Economic Studies*, 66, 83-114.

Posner, Richard A. (2003), *Economic Analysis of Law* (6th ed., Aspen Publishers).

Schankerman, Mark, and Suzanne Scotchmer (2001), "Damages and Injunctions in Protecting Intellectual Property," *RAND Journal of Economics*, 32, 199-220.